

If  $f(x) = 3x - 1$ ,  $g(x) = x - 5$ , and  $h(x) = x^2 - 3$ , find the following

1.  $(f \cdot g)(x)$
2.  $(g - f)(x)$
3.  $(g \circ h)(x)$
4.  $(f \circ g)(1)$

**Solution 1:**  $(f \cdot g)(x) = f(x) \cdot g(x) = (3x - 1)(x - 5) = 3x^2 - 15x - x + 5 = 3x^2 - 16x + 5$

**Solution 2:**  $(g - f)(x) = g(x) - f(x) = (x - 5) - (3x - 1) = x - 5 - 3x + 1 = -2x - 4$

**Solution 3:**  $(g \circ h)(x) = g(h(x)) = \underbrace{(x^2 - 3)}_{h(x)} - 5 = x^2 - 8$

**Solution 4:**  $(f \circ g)(1) = f(g(1)) = 3(\underbrace{1 - 5}_{g(1)}) - 1 = -12 - 1 = -13$

5. On the same set of axes graph the one-to one function,  $f(x) = 2x - 2$  and its inverse. Write the equation of  $f^{-1}$ .

**Solution:**

I. Write  $f(x)$  as  $y$ :  $y = 2x - 2$

II. Exchange  $x$  and  $y$ :  $x = 2y - 2$

III. Solve for  $y$ :

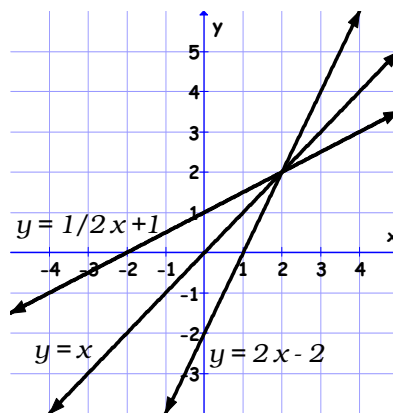
$$x = 2y - 2$$

$$x + 2 = 2y$$

$$\frac{x + 2}{2} = y$$

IV. Rewrite in terms of  $f^{-1}(x)$ .

$$f^{-1}(x) = \frac{x + 2}{2} = \frac{1}{2}x + 1$$



Use the properties of logarithms to write each expression as a single logarithm.

6.  $\log_2 6 + \log_2 5 - \log_2 3$

**Solution:**  $\log_2 6 + \log_2 5 - \log_2 3 = \log_2 \frac{6(5)}{3} = \log_2 10$

7.  $4\log x + \log(x+7) - \log x^2$

**Solution:**  $4\log x + \log(x+7) - \log x^2 = \log x^4 + \log(x+7) - \log x^2 = \log \frac{x^4(x+7)}{x^2} = \log x^2(x+7)$

8. Express  $\log_5 \left( \frac{x^3(x+5)^2}{\sqrt[3]{x-1}} \right)$  as a sum and/or difference of multiples of logarithms.

**Solution:**  $\log_5 \left( \frac{x^3(x+5)^2}{\sqrt[3]{x-1}} \right) = \log_5 x^3 + \log_5 (x+5)^2 - \log_5 (x-1)^{\frac{1}{3}} = 3\log_5 x + 2\log_5 (x+5) - \frac{1}{3}\log_5 (x-1)$

9. Use the change of base property to approximate  $\log_5 8$  to four decimal places.

**Solution:**  $\log_5 8 = \frac{\log 8}{\log 5} = 1.2920$

10. Solve for  $x$ :  $\log_3 \frac{1}{27} = x$ .

**Solution:**  $\log_3 \frac{1}{27} = x$

$$3^x = \frac{1}{27}$$

$$3^x = 3^{-3}$$

$$x = -3$$

11. Solve  $2^{(3x-1)} = 6$  for  $x$ . Give an exact solution and also approximate the solution to four decimal places.

**Solution:**  $2^{(3x-1)} = 6$

$$\log_2 6 = 3x - 1$$

$$(\log_2 6) + 1 = 3x$$

$$\frac{(\log_2 6) + 1}{3} = x$$

$$x = \frac{\frac{\log 6}{\log 2} + 1}{3} \approx 1.1950$$

Solve each logarithmic equation for  $x$ . Give an exact solution.

12.  $\log_4 x = 3$

**Solution:**  $x = 4^3 = 64$

13.  $\ln e^3 = x$

**Solution:** Property of logarithms:  $x = 3$

14.  $\log_3(4x+1) = 2$

**Solution:**  $3^2 = 4x+1$

$$9 = 4x + 1$$

$$8 = 4x$$

$$2 = x$$

15.  $\log_7 4 + \log_7 x = 1$

**Solution:**  $\log_7 4x = 1$

$$4x = 7^1$$

$$x = \frac{7}{4}$$

16.  $\log(x+1) - \log(x-2) = 2$

**Solution:**  $\log \frac{(x+1)}{(x-2)} = 2$

$$10^2 = \frac{x+1}{x-2}$$

$$100(x-2) = x+1$$

$$100x - 200 = x + 1$$

$$99x = 201$$

$$x = \frac{201}{99}$$

17.  $3^{\log_3 x} = 5$

**Solution:** Property of logarithms:  $x = 5$

18.  $\log_x 4 = 2$

**Solution:**  $x^2 = 4$ ;  $x = 2$

19. Solve  $\ln(5x - 6) = 3$  accurate to four decimal places.

**Solution:**  $e^3 = 5x - 6$

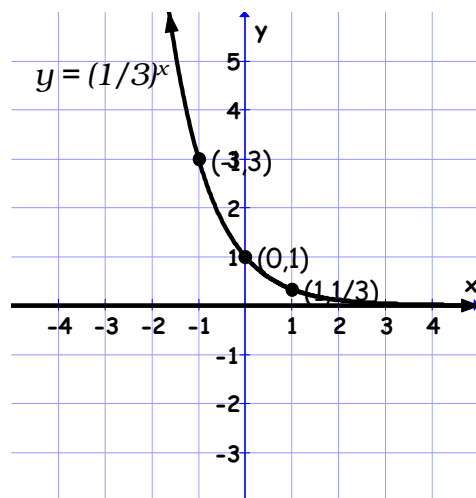
$$\frac{e^3 + 6}{5} = x$$

$$x = 5.2171$$

20. Graph the function:  $f(x) = \left(\frac{1}{3}\right)^x$ . Label three exact points and list the domain and range.

**Solution:**

$x$	$f(x) = \left(\frac{1}{3}\right)^x$
-1	3
0	1
1	$\frac{1}{3}$

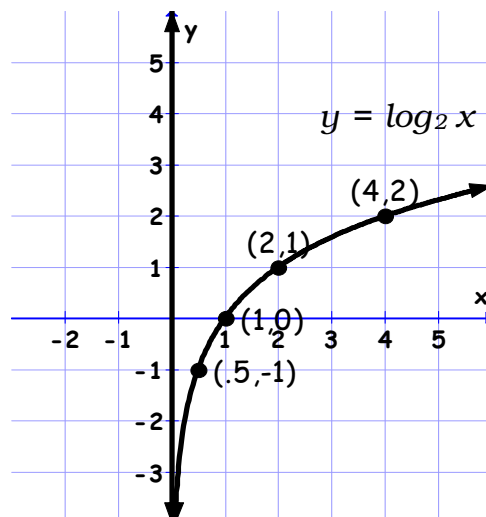


Domain:  $(-\infty, \infty)$  Range:  $(0, \infty)$

21. Graph the function  $g(x) = \log_2 x$ . Label three exact points and list the domain and range.

**Solution:**

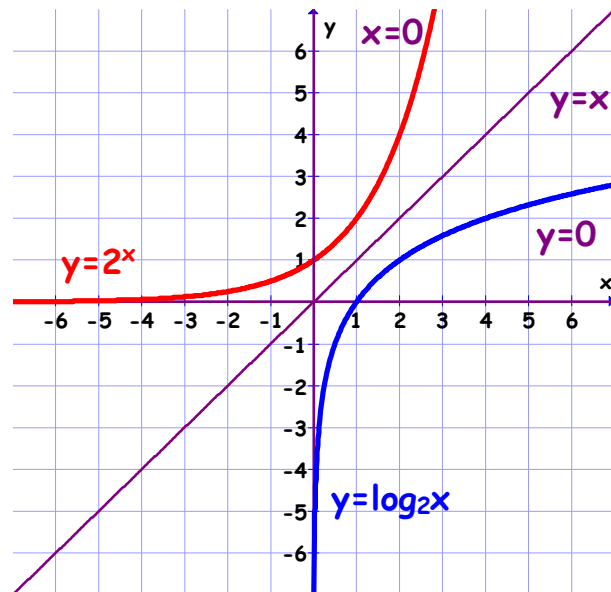
$x$	$y = \log_2 x$
$\frac{1}{2}$	-1
1	0
2	1



Observe the inverse property of the log function: The inverse of  $g(x) = \log_2(x-1)$  is:

1.  $y = \log_2 x$
2.  $x \leftrightarrow y : x = \log_2 y$
3. Solve for y:  $2^x = y$
4. Graph the exponential function:

$x$	$y = 2^x$
-1	$\frac{1}{2}$
0	1
1	2



Both graphs are symmetrical in respect of  $y = x$

The horizontal asymptote  $y=0$  changed to a vertical asymptote  $x = 0$ .

Domain:  $(0, \infty)$  Range:  $(-\infty, \infty)$

22. Determine the future value  $A = P \left( 1 + \frac{r}{n} \right)^{nt}$  of \$8,000 deposited in a certificate of deposit if it earns 6% compounded monthly after 2 years.

**Solution:** The formula for the accumulated value is:  $A = P \left( 1 + \frac{r}{n} \right)^{nt}$ , where:

$P$  = the present value,

$n$  = the number of compound periods per year,

$r$  = rate in percents,

$t$  = years.


In the example:  $P = \$8\,000$ ;  $n = 12$ ;  $r = 0.06$ ;  $t = 2$ .

$$A = 8000 \left( 1 + \frac{0.06}{12} \right)^{12(2)}$$


The correct answer is most likely to be achieved if we follow the following steps using the calculator:

Step 1: Calculate the fraction  $\frac{0.06}{12}$ .



Step 2: Click: 1 +  

Result: 1.005

Step 3: Click:  (calculate  $12 \times 2$  by hand) 24

Result: 1.127159776

Step 4: Click:  8000

Result: 9017.27821

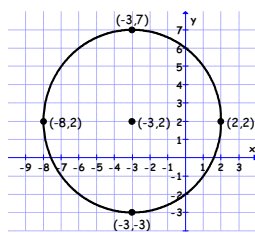
**Answer:** \$ 9017.28

23. Write the standard form of the equation of the circle whose radius is 3 and whose center is (2, -1).

**Solution:** The standard form of the equation of the circle is  $(x - h)^2 + (y - k)^2 = r^2$ , where the center is  $(h, k) = (2, -1)$  and  $r = 3$ . So:  $(x - 2)^2 + (y + 1)^2 = 9$

24. Sketch a graph of the circle:  $(x + 3)^2 + (y - 2)^2 = 25$ . Label the center and 4 exact points which lie on the circle.

**Solution:** The standard form of the equation of the circle is  $(x - h)^2 + (y - k)^2 = r^2$ , where the center is  $(-3, 2)$ , the radius is 5



25. Sketch a graph of the circle:  $x^2 + y^2 = 49$ . Label the center and 4 exact points which lie on the circle.

**Solution:** Center:  $(0, 0)$  Radius: 7

